LOCAL PARTITIONING ALGORITHMS
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1. PageRank method

DEFINITION 1.1. — *Conductance of $S \subseteq G$ is given by*

$$\Phi(S) := \frac{|\partial S|}{\min\{vol(S), vol(G-S)\}}$$

THEOREM 1.2 (Nibble). — *Let $C \subseteq G$ be any set such that $vol(C) \leqslant vol(G)/2$ and $\Phi(C) \leqslant O(\phi^2/\ln^2 m)$. There is a $C^* \subseteq C$ so that $vol(C^*) \geqslant vol(C)/2$, and if $v \in C^*$ then there exists $b \in \{1, \ldots, \log m\}$ so that* `PageRank-Nibble`(v, ϕ, b) *finds a set $S \subseteq G$ so that:*

(i) *Conductance: $\Phi(S) \leqslant \phi$,*
(ii) *Volume: $2^b/2 \leqslant vol(S) \leqslant 2/3 \cdot vol(G)$,*
(iii) *Intersection: $vol(S \cap C) \geqslant 2^b/4$*

Running time $O(2^b \ln^2 m/\phi^2)$.

THEOREM 1.3 (Partition). — *If there exists $C \subseteq G$ with $\Phi(C) = O(\phi^2/\ln^2 m)$, then with high probability* `PageRank-Partition`(ϕ) *finds a set S such that:*

(i) *Conductance: $\Phi(S) \leqslant \phi$, and*
(ii) *Volume: $vol(S) \geqslant vol(C)/2$*

Running time $O(m \ln^4 m/\phi^2)$.

2. Spielman's multi-partition

Spielman-Teng's `Multi-Partition` algorithm essentially (and roughly) guarantees:

THEOREM 2.1 (Multi-partition (roughly)). — *For every* $0 < \theta < 1$, `Multi-Partition`(θ) *outputs a multi-partition* $\{C_1, \ldots, C_k\}$ *of* G *so that*

(i) *The number of cross-cluster edges (cut size) is at most* $O(\theta m \ln^2 m)$, *and*

(ii) *For all* i, *the conductance of the induced graph on* C_i *is at most* $O(\theta^3 / \ln^3 m)$

Running time is $O(m \ln^{O(1)} m/\theta^5)$. *This can be improved slightly using* `PageRank-Partition` *as a subroutine.*